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Diversification at Financial Institutions and Systemic Crises

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Abstract

We show that the diversification of risks at financial institutions has unwelcome effects by increasing the likelihood of systemic crises. As a result, complete diversification is not warranted and the optimal degree of diversification is arbitrarily low. We also identify externalities that cause financial institutions to diversify beyond the optimal level. Recent developments in the financial system that have facilitated diversification may thus have reduced welfare.

JEL classification: G21, G28

Keywords: diversification, financial consolidation, conglomeration, securitization, systemic risk

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1 Introduction

The financial system has undergone important changes in recent years. For one, deregulation has allowed institutions to expand beyond their traditional business boundaries. This has led to the emergence of *conglomerates* which combine banking, securities, and insurance activities in one organization. Waves of mergers and acquisitions have led to the *consolidation* of the financial industry, creating institutions that operate on a global level. Moreover, ongoing *securitization* has allowed institutions to transfer a wide range of risks, which previously had to be held on their balance sheets.

These changes have facilitated the diversification of risks in the financial system. For example, conglomeration creates functional diversification in that a previous commercial bank now also engages in securities and insurance activities. Consolidation fosters in particular geographical diversification, as mergers may combine institutions from different countries or regions. Securitization has enabled institutions to shed parts of their specific risk. For instance, the various new methods for credit risk transfer have made it possible to reduce regional or industry specific risk at banks.

Portfolio theory (Markowitz, 1952) suggests that this development should be welcomed as diversification reduces the risks at financial institutions and thus makes their failure less likely. Not surprisingly, therefore, diversification gains are usually seen as a main benefit of the recent changes in the financial system. For example, the G10 report (Group of Ten, 2001) on financial consolidation writes: "The one area where consolidation seems most likely to reduce firm risk is the potential for diversification gains". Similarly, the BIS report on credit risk transfer (BIS, 2004) states that "Innovation in financial markets, and within that the development of new financial instruments such as credit derivatives, is generally to be welcomed as ...enabling better diversification... ."

By contrast, this paper shows that diversification of risks in the financial system is not necessarily desirable. The reason is that, first, although diversification reduces risks at each individual institution, from the viewpoint of the financial system it only causes a reallocation of risks. Second, as diversification leads to the sharing of risks across institutions, it has the effect of making their exposures more similar to each other. For example, two commercial banks, one previously U.S. focused and the other focused on Europe, may due to global consolidation become exposed to the same risks, both on the asset and the liability side of their balance sheets. Similarly, functional diversification of banks into the insurance business (and, vice versa, of insurance companies into banking activities) creates institutions that now face like risks.[1]

This increase in the similarities among institutions unambiguously raises the likelihood of systemic crises (that is, crises where institutions are failing together). Intuitively, this is because a shock that was previously experienced only in one part of the financial system, now affects more institutions and can, if sufficiently large, also lead to their failure. When systemic crises are more costly than individual crises,[2] diversification has to trade-off this effect against the benefits of diversification (which come in the form of reducing the overall likelihood of an institution failing). We show that, as a result, complete diversification of risks in the financial system is undesirable. The optimal degree of diversification can be arbitrarily small, depending on the cost of a systemic crisis relative to individual crises.

These results are obtained in the absence of contagion effects. Allowing for contagion reduces the case for diversification further. The reason is that an institution is more likely to suffer from spillovers when it is in difficulties itself. The increase in similarities due to diversification hence facilitates contagion by making it more likely that if one institution fails, other institutions are in difficulties at the same time. This effect can be even so pronounced that diversification increases the overall probability of an institution failing.

Rational institutions should of course anticipate these effects, suggesting that they restrict diversification to the desirable amount. However, we show that this is not the case due to the presence of a diversification externality. The latter arises because diversification at an institution increases its similarity to other institutions, which is undesirable for the latter as it makes crises more costly for them and enhances contagious spillovers.

[1] Recent evidence suggest that financial institutions, at least the very large ones, have indeed become more similar: the correlation of the share prices of large U.S. banks increased from 28% to 54% between 1995 and 2000 (Group of Ten, 2001).

[2] In our analysis this is because a financial institution suffers larger costs when it faces a crisis jointly with other institutions, as it is then more difficult to liquidate assets at their fair value.

For example, a commercial bank that starts insurance activities may reduce welfare in the insurance industry by making it more likely that it will draw upon a common pool of liquidity when the insurance sector has liquidity needs. Because of this externality, financial institutions diversify more than the socially optimal amount. A straightforward implication is that the diversification brought about by recent developments in the financial system may have been undesirable, as it may have pushed the level of diversification further beyond its optimal level.

Regulators have typically embraced diversification on the grounds that it reduces the failure risk of an institution. By contrast, our analysis suggests that regulation should rather discourage financial institutions from diversifying in order to offset the diversification externality. However, in evaluating this and the other results it should be emphasized that our analysis keeps constant the total amount of assets and liabilities held by fragile institutions. If diversification leads net to risk being shifted out of relatively more fragile institutions or if institutions diversify into new activities that are not correlated with existing activities in the financial system, there is still a case for encouraging diversification.

1.1 Related Literature

The present paper relates to several contributions that have pointed at undesirable effects of diversification. Winton (1999) has shown that diversification at banks may produce endogenously higher risks. This is because following diversification, bank managers have less incentives to monitor loans. Acharya, Hasan and Saunders (forthcoming) provide empirical evidence for such an effect. In Wagner (2006) banks feel safer following diversification and increase their risk-taking, which can reduce overall welfare. A common feature of these papers is that diversification increases total asset risk in the financial sector, while in the present paper the assets of the financial sector do not change.[3] Goldstein and Pauzner

[3]Several papers have also emphasized that banks have incentives to choose correlated assets. Acharya and Yorulmazer (2004) present a model where such herding in investment choices arises because banks want to increase the likelihood of failing jointly in order to induce a regulator to bail them out. In Acharya and Yorulmazer (2005), bank owners invest in correlated assets because they do not internalize the costs of a joint failure due to limited liability. By contrast, in the present paper banks dislike being correlated

(2004) show that diversification by investors can cause contagion across countries, arising from a wealth effect. They also provide an example for that full diversification can actually be inferior to partial diversification. The present paper differs from their work in that we do not rely on contagion effects. Rather, diversification is costly because it makes the joint failure of institutions more likely.

The effect of diversification in our paper builds closely on an insight provided by Shaffer (1994), who has shown that mergers increase the likelihood of the joint failure of institutions. Intuitively, this is because a merged entity also suffers from shortfalls at the other entity, which creates spillover effects similar to the ones that arise for diversification into the activities of other institutions.

Diversification is in practice often achieved through the transfer of risk from one to another institution. Allen and Gale (2005) show that a transfer of risks from the banking into the insurance sector can add to systemic risk when regulation is ill-designed. In Allen and Carletti (2006) risk transfer among sectors creates contagion by subjecting the banking system to the systemic risk of the insurance sector. In Wagner and Marsh (forthcoming), risk transfer has stability effects when it takes place among institutions that differ with respect to their fragilities. While total risks may stay the same in these papers, they differ from the present one in that our results arise independently of suboptimal regulation, contagion effects or risk shifting among institutions of different fragilities.

The present paper also relates to the ongoing discussion on the welfare effects of financial integration and the desirability of interlinkages among financial institutions. Several contributions have shown that interlinkages can be destabilizing because they may cause a financial crisis to spill over to other financial institutions (e.g., Rochet and Tirole 1999, Allen and Gale 2000, Aghion, Bolton and Dewatripont 2000, Freixas, Parigi and Rochet 2000). However, recent papers have emphasized that, overall, such linkages cannot reduce institutions' welfare when the spillovers are anticipated. This is for the simple reason that if linkages are not beneficial, financial institutions will choose not to be connected (Kahn and Santos, 2005, and Brusco and Castiglionesi, 2006). This result does not hold in the

with other institutions. A higher correlation across banks arises only indirectly as an unwelcome side effect of diversification.

setting of this paper. While diversification (which can be interpreted as an interlinkage between financial institutions because it could be achieved through cross-holdings of assets or liabilities) may be optimal for an institution, it lowers welfare at other institutions due to the diversification externality. As a result, the possibility for financial institutions to form interlinkages can be undesirable, even though their consequences are fully anticipated.

The paper proceeds as follows. The next section describes the model. Section 3 studies the impact of diversification. Section 4 analyzes the regulatory implications of our analysis. The final section concludes.

2 The Model

In our analysis we focus on the institutional structure of the banking sector, for which stability issues are probably most prevalent. The economy consists of two banks, which are specialized in different activities. Each bank has collected one unit of funds from investors, of which a share d is in the form of deposits and a share $1-d$ is capital. We take this capital structure initially as given, which may, for example, be the result of binding capital requirements. Both shareholders and depositors are risk neutral and their required return is zero.

There are three dates. At date 1, each bank invests its funds into the activity it is specialized in. This specialization may be given various interpretations. For example, it may refer to specialization into different industries or regions, to specialization into wholesale and retail business or to specialization into commercial and investment banking activities. For concreteness, assume that bank 1 invests in an asset X and bank 2 in an asset Y. The corresponding asset returns, x and y, are independently drawn and are uniformly distributed on $[0,1]$. Their density is hence given by $\phi(x)=\phi(y)=1$. Both assets are assumed to be illiquid in that they only mature at date 3 and cannot be (physically) liquidated before that date.

At date 2, the asset returns become known. The value of the portfolio of bank 1 and 2 is then $v_1=x$ and $v_2=y$, respectively. As will be shown below, runs may occur on a bank for a low value of its portfolio, forcing the bank to sell its asset to investors.

We assume two features of such asset sales. First, an asset can only be sold at a discount to its true value. This may be, for example, because of *fire-sale* prices (arising from the immediacy of selling in a run), informational problems (due to buyers being less informed about the asset), outsiders being inferior users of the asset, or because investors require compensation for holding illiquid and risky assets. Second, the discount rises with the overall amount of assets sold by banks. Such an increasing discount arises from 'cash-in-the-market pricing' (e.g., Allen and Gale, 2004, Schnabel and Shin, 2004, and Gorton and Huang, 2004): when investors only hold limited liquidity, asset sales have an impact on their overall liquidity and thus raise investors' required returns. Moreover, it may arise when investors' risk-bearing capacity is limited or when informational problems are intensified when banks sell more assets (banks will typically sell first the informationally less sensitive part of their portfolios). An inelastic demand is also a typical cornerstone of market microstructure models.

We model these characteristics in reduced form by assuming that banks face a residual inverse demand function for their assets. These demand functions are denoted p_x and p_y and take the form $p_x(s) = x - c(s)$ and $p_y(s) = y - c(s)$. $c(s)$ (with $c(s) \geq 0$ and $c'(s) > 0$) denotes the (average) cost of selling a unit of the asset when total asset sales in the economy are s. s consist of the sum of the asset sales at bank 1 (s_1) and at bank 2 (s_2), with $0 \leq s_{1,}s_2 \leq 1$ (if $s_i = 1$, bank i ($i = 1, 2$) completely sells its asset). We also assume that any surplus arising on the buyer side is fully extracted by banks (for example, because investors submit a complete demand schedule).[4]

Example 1: *Suppose that there is only a single investor, who has quadratic utility with risk aversion parameter γ. Assume that when the investor buys the asset at date 2, he faces uncertainty about its return at date 3. This may be, for example, because the bank cannot perfectly communicate the asset value or because the investor has inferior monitoring capabilities which cause an increase in the riskiness of the asset. This uncertainty is assumed to take the form of an additive shock z (common to both*

[4]This assumption is not crucial. What is important for our welfare analysis, however, is that there is an overall deadweight loss in the asset sale.

assets) at date 3.[5] *Shock z is normally distributed with zero mean and variance σ^2. The investor's expected utility from buying amounts of s_1 and s_2 of assets X and Y from bank 1 and 2 is then*

$$u(s_1, s_2) = s_1(x - p_x) + s_2(y - p_y) - \gamma(s_1 + s_2)^2\sigma^2/2$$

Setting $u(s_1, s_2)$ to zero (because the surplus is fully extracted) it follows that $c(s) = c_0 s$, where $c_0 = \gamma\sigma^2/2$ (in line with this example, Grossman and Miller (1988) provide an explanation for 1987 crash based on risk-averse traders that need to be compensated through higher returns).

Whether or not a bank run occurs at date 2, depends on the value of a bank's portfolio and on depositors' beliefs about the actions of other depositors. Generally, there are three situations. When portfolio values are sufficiently high, bank runs will occur independently of depositors' beliefs. When they are sufficiently low, bank runs cannot occur. In an intermediate range of portfolio values, the incidence of a run depends on depositors' beliefs. In order to demonstrate the main arguments in their most parsimonious way, we assume that beliefs are such that runs never occur in this range. This will be the case when depositors believe that other depositors will never run, whenever this may be rational. However, our results do not depend on this particular equilibrium selection, as we will discuss later.

A run on bank i will then occur exactly if the value of the bank's portfolio is lower than the value of its deposits ($v_i < d$).[6] To see this, suppose first that $v_i < d$. The date 3 value of the bank's asset is then lower than the value of the deposits. Depositors at the bank thus anticipate that they will not be paid in full at date 3. Because of the first-come-first-serve

[5] There is, of course, likely to be idiosyncratic uncertainty as well. Introducing the latter would have the effect of making the discount for an asset also explicitly contingent on how much of this asset is sold (in our analysis the asset sale discount is only implicitly contingent on it through its dependence on the total asset sales in the banking sector).

[6] Thus, runs occur here at insolvent banks. Our results also hold (and are in a sense even stronger) for runs at solvent banks (see Section 3.1, 'Other beliefs and contagion'). Note that even in the presence of a regulator, insolvency runs can occur because of regulatory forbearance, asymmetric information about bank assets or simply because a run anticipates closure by the regulator.

nature of deposits, it is then individually optimal for these depositors to run immediately on the bank at date 2. Hence, all depositors at bank i run.

Suppose next that $v_i \geq d$. Deposits can then be paid back in full at date 3 if no run occurs at date 2. Believing that depositors will not run at date 2 is then a rational belief. Hence, a depositor does not run on the bank (assuming a weak preference for not running).

When a run occurs, a bank has to liquidate its asset in full to meet depositors' demands, as $v_i < d$. If there is a run at one bank only (individual crisis), the asset sale discount will be $c(1)$ as we have then $s = s_i = 1$. If there is a run on the other bank as well (systemic crisis), the discount rises to $c(2)$, as $s = s_1 + s_2 = 2$. Since a bank is always insolvent when a run occurs, it will cease to exist after having sold its asset. If there is no run, the bank survives and returns v_i at date 3 to its depositors and shareholders.

Figure 1 illustrates the different outcomes. In area A we have $x \geq d$ and $y \geq d$. Hence $v_1(x), v_2(x) \geq d$ and there is no run at either bank. Note that since densities $\phi(x)$ and $\phi(y)$ are 1, areas in Figure 1 correspond to probabilities. It follows that the probability of no bank run occurring is $(1-d)^2$. In area B we have systemic crises since $x < d$ and $y < d$, occurring with probability d^2. In area C we have individual crises at bank 1 as $x < d$ but $y \geq d$, while in area D we have individual crises at bank 2. Thus, an individual crisis occurs at a bank with probability $d(1-d)$.

As depositors and shareholders are risk neutral, a bank's objective is to maximize the sum of its expected return to depositors and shareholders (depositors have to break even on average, therefore, any impact on their payoffs is internalized by the bank). This return is given by the expected return on the bank's asset minus the expected costs from having to sell the asset at date 2. As the asset return is given (equal to 1/2), banks simply minimize the expected costs of asset sales. Denoting the probability of an individual and a systemic crises at bank i by π_i^I and π^S (note that $\pi_i^S = \pi_j^S = \pi^S$ by definition), these costs are

$$C_i = \pi_i^I c(1) + \pi^S c(2) \tag{1}$$

Defining with $q = c(2)/c(1)$ the relative cost of a systemic crisis compared to an individual crisis (with $q > 1$), C_i can be written as

$$C_i = c(1)(\pi_i^I + q\pi^S) \tag{2}$$

Example 1 continued: *From $c(s) = c_0 s$, we have $q = c(2)/c(1) = 2$. Hence the of cost of a systemic crisis is twice as large as the cost of an individual crisis.*

Overall welfare in the economy consists of the sum of the expected returns on both banks (we can ignore the investors that acquire assets in a bank run since any surplus is extracted). It follows that welfare maximization requires to minimize the expected efficiency losses from asset sales in the economy

$$C = C_1 + C_2 = c(1)(\pi_1^I + \pi_2^I + 2q\pi^S) \tag{3}$$

3 The Impact of Diversification

As the two assets are uncorrelated, there are diversification benefits to exploit.[7] To study the effect of diversification, we now allow banks to invest at date 1 also in the activity they are not specialized in (equivalently, banks may be still constrained to source their own activities, but may subsequently trade assets with each other). We denote with $r_i \in [0, 1/2]$ the share of funds bank i invests into its non-specialized activity, that is its degree of diversification. For $r_i = 1/2$, the bank has identical exposures to both activities, i.e., the bank is fully diversified. When $r_i = 0$, the bank remains undiversified. For most of the analysis it is convenient to work with a monotonic transformation $\widetilde{r}(r) = r/(1-r)$ of r, where $\widetilde{r} = 0$ and $\widetilde{r} = 1$ refer to no and full diversification, respectively.

Using these definitions, the portfolio values of bank 1 and 2 can be written as

$$v_1(x, y) = (1 - r_1)x + r_1 y = \frac{1}{1 + \widetilde{r}_1}x + \frac{\widetilde{r}_1}{1 + \widetilde{r}_1}y \tag{4}$$

$$v_2(x, y) = r_2 x + (1 - r_2)y = \frac{\widetilde{r}_2}{1 + \widetilde{r}_2}x + \frac{1}{1 + \widetilde{r}_2}y \tag{5}$$

In the previous section we have shown that a bank run occurs at bank i when v_i is below d. From this we can derive for each bank the minimum return y that is needed to avoid a run for a given realization x. From setting $v_i = d$ in (4) and (5) these minimum returns are

$$y_1(x) = (1 + 1/\widetilde{r}_1)d - (1/\widetilde{r}_1)x \text{ and } y_2(x) = (1 + \widetilde{r}_2)d - \widetilde{r}_2 x \tag{6}$$

[7] We model here diversification on the asset side, but our main arguments should also apply to diversification on the liability side (for example, arising from the acquisition of deposits in other regions).

e.g. bank 1 fails if $y < y_1(x)$ and bank 2 fails if $y < y_2(x)$.

Figure 2 shows these minimum return functions. The intersections with the axes are given by

$$x_1(0) = (1 + \widetilde{r}_1)d;\ x_2(0) = (1 + 1/\widetilde{r}_2)d;\ y_1(0) = (1 + 1/\widetilde{r}_1);\ y_2(0) = (1 + \widetilde{r}_2)d \qquad (7)$$

In area A we have $y \geq y_1(x)$ and $y \geq y_2(x)$ and hence no bank fails. In area B y is below the minimum returns for both banks, hence there is a systemic crisis ($y < y_1(x)$ and $y < y_2(x)$). In are C we have an individual crisis at bank 1 as $y < y_1(x)$ and $y \geq y_2(x)$ and in area D there is an individual crisis at bank 2 since $y \geq y_1(x)$ but $y < y_2(x)$.

Figure 3 shows next the effect of moving from an undiversified banking sector to a banking sector with a symmetric degree of diversification $\widetilde{r}$ ($\widetilde{r} = \widetilde{r}_1 = \widetilde{r}_2$). As shown, this causes a rotation of $y_1(x)$ counter-clockwise around the point (d, d). Intuitively, this is since the bank is now more exposed to y and less exposed to x, the bank is more likely to fail in areas where $y > x$ and less likely to fail in areas where $x < y$. At $x = y$, diversification does not matter. Conversely, $y_2(x)$ rotates clockwise around (d, d).

The rotation of $y_1(x)$ and $y_2(x)$ has the effect of reducing the probability of an individual crisis at bank 1 and 2 in areas D and B, respectively. In these areas, a bank failed previously because its asset suffered a low return but now survives because it bought into another assets that performs better. However, it also increases the probability of systemic crises at both banks (areas A and C for each bank). For example, in area A bank 2 now fails because it bought into the asset of bank 1. This implies that bank $1's$ asset must have a very low return because bank 2 previously survived. Therefore, bank 1 also fails in the same area and there is a systemic crisis.

We next analyze the impact on the overall probability of a crisis at bank 2 (because of symmetry, results also hold for bank 1). We focus on parameter constellations for which $y_1(0) < 1$ and $x_2(0) < 1$, which is the situation depicted in the Figures.[8] From Figure 2

[8] Using (7), these conditions translate into a lower bound on the degree of diversification: $\widetilde{r} > d/(1-d)$ (which can be made non-binding for sufficiently small d). If this condition is not fulfilled, the reduction in individual crises (areas D and B) would be truncated by the distribution function, having the effect of *lowering* the gains from diversification.

we have that the probability of an individual crisis is given by

$$\pi_2^I = \frac{1}{2}(x_2(0) - d)d - \frac{1}{2}(x_1(0) - d)d = d^2(\frac{1}{2\widetilde{r}_2} - \frac{\widetilde{r}_1}{2}) \tag{8}$$

and the probability of a systemic crisis is

$$\pi^S = d^2 + \frac{1}{2}(y_2(0) - d)d + \frac{1}{2}(x_1(0) - d)d = d^2(1 + \frac{\widetilde{r}_1 + \widetilde{r}_2}{2}) \tag{9}$$

Hence, the overall probability of a crisis at bank 2 is

$$\pi_2 = \pi_2^I + \pi^S = d^2(1 + \frac{1}{2\widetilde{r}_2} + \frac{\widetilde{r}_2}{2}) \tag{10}$$

Taking derivative with respect to $\widetilde{r}$ gives $\partial\pi_2/\partial\widetilde{r} = \partial\pi_2/\partial\widetilde{r}_2 = d^2(-1/\widetilde{r}_2^2 + 1)/2$, which is negative for $\widetilde{r}_2 < 1$. Thus, the probability of a crisis is reduced, which is the effect usually attributed to diversification. However, this does not imply that welfare increases since the probability of a systemic crisis has risen: from (9) we have $\partial\pi^S/\partial\widetilde{r} = \partial\pi^S/\partial\widetilde{r}_1 + \partial\pi^S/\partial\widetilde{r}_2 = d^2 > 0$. In fact, in a fully diversified banking sector systemic crises are twice as likely as in the undiversified banking sector (from (9) we have that $\pi^S(\widetilde{r} = 1)/\pi^S(\widetilde{r} = 0) = 2$).

This raises the question of whether optimal diversification is perhaps incomplete, contrary to what portfolio theory would suggest. The optimal degree of diversification, denoted $\widetilde{r}^*$, minimizes $C_1 + C_2$ (equation 3), which simplifies here to minimizing $c(1)(\pi_2^I + q\pi^S)$ due to symmetry. Using (8) and (9) we can then find

$$\widetilde{r}^* = \frac{1}{\sqrt{2q - 1}} \tag{11}$$

Recalling that $q > 1$, we have that $\widetilde{r}^* < 1$. Hence, diversifying completely is not optimal. What is the reason for this result? From $\partial\pi_2/\partial\widetilde{r}$ we have the marginal benefits from diversification (in terms of reducing the probability of crises) fall with the degree of diversification and become zero for full diversification ($\partial\pi_2/\partial\widetilde{r} = 0$ at $\widetilde{r} = 1$), which is also a result of portfolio theory. By contrast, the marginal costs of diversification (in terms of increasing the probability of a systemic crises) are always positive since rotations in the minimum return functions enlarge areas A and C in Figure 3 (in fact, because of the uniform distribution the marginal costs are constant here: $\partial\pi^S/\partial\widetilde{r} = d^2$). Therefore, for a sufficiently high degree of diversification, marginal costs always exceed the benefits, causing the optimal degree to be below full diversification.

Proposition 1 *Optimal diversification is incomplete.*

Note that this result arises only because a systemic crisis is more costly than an individual crisis: for $q = 1$ we would get $\widetilde{r}^* = 1$ from equation (11). By contrast, the optimal degree of diversification becomes arbitrarily small for large q ($\lim_{q->\infty} \widetilde{r}^* = 0$)

Example 1 continued: *Since $q = c(2)/c(1) = 2$, we obtain from (11) (after transformation into r) a socially optimal degree of diversification of $r^* = 0.37$, which is less than full diversification ($r = 0.5$).*

Rational institutions should anticipate these effects and limit diversification to the extent that is desirable. This suggests that an increase in banks' diversification possibilities should not reduce welfare. However, this argument presumes the absence of externalities.

To study whether there are externalities, consider the impact of more diversification at bank 1 (an increase in r_1) on bank 2. Figure 3 has shown that this leads to a rotation of $y_1(x)$. This has no impact on bank $2'$s probability of a crisis, since its failure is determined by $v_2 < d$. However, it changes the amount of systemic crises that bank 2 experiences: in area A the bank faces now systemic crises instead of an individual crises (formally, from (11) we have $\partial \pi_2^S / \partial r_1 = d^2/2 > 0$). The reason is that by diversifying, bank 1 becomes more similar to bank 2, which increases the likelihood that when bank 2 fails, bank 1 fails at the same time. As systemic crises are more costly than individual crises, there is hence a negative externality of diversification. It follows that

Proposition 2 *The equilibrium degree of diversification in the banking sector exceeds the socially optimal one.*

Hence increased diversification possibilities in the banking sector may reduce welfare, as they may push diversification further beyond $\widetilde{r}^*$. Note that because of symmetry we have in equilibrium that diversification at bank 1 equals diversification at bank 2. It follows from (4) and (5) that $v_1(x, y) + v_2(x, y) = x + y$, i.e., the total asset holdings of the banking sector are unchanged.[9]

[9]Thus, the diversifications effects also arise when diversification is achieved through assets trade at date 1 (however, in order to obtain an externality, one would obviously need more than one bank per asset)

The equilibrium degree of diversification (in absence of any impediments), denoted $\widetilde{r}^M$, can be found through solving the maximization problem for bank 2: $\widetilde{r}_2^M = \arg\min_{\widetilde{r}_2} \pi_2^I + q\pi^S$ and setting $\widetilde{r}_1^M = \widetilde{r}_2^M = \widetilde{r}^M$ because of symmetry. We obtain

$$\widetilde{r}^M = 1/q^{1/3} \tag{12}$$

Example 1 continued: *The equilibrium degree of diversification is* $r^M = 44\%$ for $q = 2$, *exceeding the optimal degree of* $r = 37\%$.

3.1 Discussion

Endogenous capital structure. The analysis has taken as given the capital structure of banks. In Appendix A we model banks' choice of their capital structure. A rationale for issuing deposits arises as deposits mitigate agency problems in banks. Following Calomiris and Kahn (1991), this is because the threat of a run created by deposits has a disciplining effect on bank managers. The optimal capital structure trades off the arising efficiency gains with the costs of bank runs.

Diversification has then additional effects, beyond the ones already described, through its impact on banks' capital structures. As diversification makes the failure of a bank less likely, it reduces the threat of a bank run. This induces banks to substitute equity with deposits to offset this effect. The resulting increase in bank fragility makes diversification less desirable than under a fixed capital structure.

Other beliefs and contagion. For expositional clarity, we have rather arbitrarily assumed that beliefs are such that in the intermediate range runs never occur. Appendix B shows that the main results carry over for what are essentially diametrical beliefs, i.e., beliefs that are such that runs always occur in the intermediate range.[10] A new element in the analysis, however, is that there can now be contagion. This is because a bank's minimum return function becomes conditional on whether a run takes place at the other bank. A failure at one bank can, therefore, spill over to another bank.

[10] The arising equilibrium has theoretic appeal, as it can be rationalized through common equilibrium selection strategies, such as risk dominance (Harsanyi and Selten, 1988) and global games (Carlsson and van Damme, 1993).

Interestingly, this worsens the case for diversification. One may have conjectured that diversification should be more beneficial when contagion effects are present; the likely argument being that diversification at bank 1 should then benefit bank 2 because it reduces the failure probability of bank 1 and thus the scope for contagion. However, this reasoning ignores that an institution is more likely to suffer from contagion when it is in a critical situation itself. Diversification creates then an effect that is comparable to the one that arises in the absence of contagion: by making banks more similar, it becomes more likely that when one bank fails, the other bank is in difficulties as well. Contagious spillovers hence increase. As a result, as Appendix B shows, the optimal degree of diversification is lower than without contagion. Furthermore, it does not hold anymore that diversifying always reduces the probability of bank failure: it is shown that when the degree of diversification is sufficiently large, further diversification increases the failure risk at banks.

Other distribution functions. Our results are not specific to the uniform distribution. In particular, Appendix C shows that Proposition 1 and 2 also hold when x and y are (independently) distributed according to an arbitrary distribution function $\phi(\cdot)$ which has full support on $[0, 1]$ (i.e., $\phi(x) = \phi(y) > 0$ for $x, y \in [0, 1]$).[11]

Diversification into new activities. We have presumed that banks diversify into risks which are already present in the financial sector (in our model because they invest into the activity of the other bank). Our results would be different if banks could diversify into new risks. Suppose, for example, that bank 2 can invest into an activity Z, which is uncorrelated with X and Y. Then, diversification gains could be reaped without making banks more similar. However, we would then be concerned with the combined effect of diversification and the introduction of a new asset. Furthermore, recent trends in the financial system (i.e., conglomeration, consolidation and securitization) relate to diversification of existing risks, rather than the creation of new activities.

Social externalities. While our analysis is based on interbank externalities, it is typically

[11] Another issue is the impact of assuming uncorrelated assets. Most activities in the financial system are probably positively correlated. Introducing positive correlation would have the effect of reducing both the gains from diversification (as diversification relies on assets not being correlated) and its costs (as institutions are then already similar in absence of diversification) but should not affect our main results.

also presumed that banking failures cause costs outside the banking sector. For example, the failure of a bank may reduce welfare in the economy because the firms it has financed have to switch to another bank, causing deadweight losses. As with our liquidation costs, such social costs of the failure of a bank are likely to be increasing in the total number of banks failing. For example, this may be because when many banks are failing, it becomes more difficult for firms that are reliant on bank financing to switch to other banks. An alternative story for increasing social costs is that regulators, who may have to compensate investors for incurred losses, face marginal costs of public funds that are increasing (see Acharya and Yorulmazer, 2004). Since diversification increases the likelihood of systemic crises, its optimal degree (in terms of minimizing these social costs) is hence likely to be incomplete as well, for the same reasons that apply for interbank costs.

Risk averse agents. In our analysis there is "risk aversion" in the sense that a lower variance of a bank's portfolio is valued by investors as it reduces the likelihood that the portfolio value falls below the threshold that triggers a run. Introducing risk aversion explicitly through agents' utility functions is not expected to change the main results. Diversification should then give rise to two additional, and opposing, effects. First, portfolio theory implies that diversification reduces the variance of bank returns absent liquidation costs. Second, diversification raises the return variance by making it more likely that extreme outcomes (that is systemic crises) arise. The net effect is not obvious and will depend on the parameters of the model (for example, for sufficiently large liquidation costs, the second effect should dominate).

Still, Proposition 1 should hold for similar reasons as before: the marginal gains from diversification in terms of reducing the portfolio variance decrease with the degree of diversification and become zero at complete diversification. By contrast, diversification always induces costs (both in terms of lowering expected returns and raising the variance of returns) by making systemic crises more likely. Thus, complete diversification should not be optimal. Furthermore, there is still a negative diversification externality, and thus Proposition 2 should be intact. This is because the only effect an increase in diversification at bank 1 has on bank 2 is that it increases the likelihood that bank 2 fails jointly with bank 1, rather than alone. This is costly for bank 2 in terms of lowering expected returns (as already shown) but also in terms of increasing variance by raising the probability of extreme outcomes.

Other interactions among institutions. A crucial feature of our model is that it is costly for a bank to be in difficulties at a time when other banks are failing or are in difficulties as well. While we have modelled this through the cost of asset sales, there are a variety of other channels. For example, there may be informational spillovers from the failure of a bank, driving up the cost of borrowing at other banks (or causing contagious runs). Or, bank failure may shrink the common pool of liquidity (Diamond and Rajan, 2005). Liquidity may also be reduced because banks borrow more when they are in difficulties. Furthermore, there may be network externalities, such as from a less well functioning payment and settlement system or through the interbank market. Although all these externalities are present regardless whether a bank is in good shape or not, their impact on a bank in difficulties will be more severe.

Non-bank institutions. Our analysis has focused on banks. This is for the reason that the banking sector is typically seen as the most crisis prone. However, the main elements of our analysis are that institutional failure is costly (in our model because of the liquidation costs) and that it is more costly for an institution to be in difficulties at times of general stress. These characteristics are not specific to banks. With some qualifications, our analysis may hence also apply to other financial institutions, such, for example, insurance companies or hedge funds. Their failure induces likewise costs, for instance because assets can only be liquidated at an inferior price. Moreover, they are also likely to suffer from being in a crisis jointly. For example, because they rely on a common pool of liquidity they will find it more difficult to liquidate assets at a fair price (as in our analysis) or to borrow funds in crisis times.

Our analysis looks in particularly suited to apply to institutions that use trading strategies which involve selling assets when their price declines. Such selling arises, for example, from dynamic hedging or program trading (e.g., because of a stop loss limit) and has been shown to cause excess price volatility (in particular, it can lead to *liquidity black holes*, see Morris and Shin, 2004). Our analysis suggests that diversification may be costly in the presence of such strategies by making it more likely that many traders sell at the same time. The resulting increase in selling pressure should intensify price movements and may trigger further rounds of selling.

Interbank lending. In our analysis there is no lending among banks, for the simple reason that banks do not hold liquidity (in Appendix A we argue that not holding liquidity may indeed be optimal when there are agency problems in the bank). Abstracting from lending is no issue for the first set of beliefs we considered: there only insolvent banks fail and thus lending is not feasible anyway. However, for other beliefs, in particular the ones analyzed in Appendix B, bank runs also occur at solvent banks. Then, some individual bank crises may be avoided from the outset, as a bank in troubles may be able to borrow from a healthy bank.[12] As the gains from diversification come solely through reductions in individual crises, this should have the effect of making diversification less beneficial.

Regulatory bail-outs and competitive effects. Regulators may be more inclined to bail out when banks fall jointly (Acharya and Yorulmazer, 2004). If this is the case, banks' incentives to diversify beyond the socially optimal level should intensify, as diversification increases similarities and thus the probability of a joint failure. Such incentives, however, may be offset by a 'last-bank-standing-effect' (Perotti and Suarez, 2002): if a bank is a sole survivor, it may be able to capture the other bank's market share. As diversification reduces the area where a bank survives alone (areas C and D in Figure 2), this will mitigate banks' diversification incentives.

Limited liability and managerial incentives. In our analysis, there is no conflict between shareholders and depositors, i.e., bank diversification is set to maximize the total value of the bank. If shareholders decide on the amount of diversification, and if they do not internalize the effects on depositors, equilibrium diversification should rise further beyond the efficient level, as shareholders do not suffer the higher liquidation costs that arise in a systemic crisis. A similar argument applies if bank diversification is under the control of bank managers because their pay-off may be independent of whether or not a bank fails jointly with other banks (a bank manager is likely to lose his job in both cases).

[12] Allowing for borrowing from outside the banking sector would reduce the scope for runs at solvent banks altogether but such lending may be restricted in crisis times, for example, because of informational problems.

4 Diversification and Financial Regulation

In this section we discuss the regulatory implications of our analysis. To start with, Proposition 2 suggests that recent deregulation may have had detrimental side effects. This is because deregulation has increased the diversification possibilities of financial institutions and thus may facilitated excessive diversification.

A related issue is whether the diversification incentives that arise from current regulation are efficient. While the more advanced of banks' risk management systems nowadays take into account correlations in various dimensions (e.g., across assets within one portfolio or across business lines), the regulatory stance towards diversification is mixed. In particular, the new Basel accord allows a diversification relief for operational risk but not for credit risk (BIS, 2005).[13] This reluctance to account for diversification in full has been a common criticism by practitioners (e.g., FleetBoston Financial, 2003).

An immediate corollary of Proposition 2 is that any diversification relief is unwanted. It would increase banks' incentives to diversify and thus further widen the gap between equilibrium diversification and the socially optimal one. Rather, since market diversification is excessive, regulators should discourage diversification:

Proposition 3 *Financial Regulation should set incentives for reducing diversification in the banking sector.*

Interestingly, this result also holds when the degree of diversification in the banking sector is actually low, for example, because achieving diversification is costly for banks. To see this, suppose that in order to implement a level of diversification $\widetilde{r}_i$, bank i has to incur costs $\tau(\widetilde{r}_i) > 0$ (arising for instance because diversification causes a loss in focus). The socially optimal degree of diversification minimizes total costs in the banking sector $C_1 + \tau(\widetilde{r}_1) + C_2 + \tau(\widetilde{r}_2)$. The first order condition for $\widetilde{r}_i$ thus writes $\partial C_i/\partial\widetilde{r}_i + \partial\tau_i/\partial\widetilde{r}_i + \partial C_j/\partial\widetilde{r}_i = 0$. Since $\partial C_j/\partial\widetilde{r}_i > 0$ (because of the negative diversification externality identified in the previous section), we have that $\partial C_i/\partial\widetilde{r}_i + \partial\tau_i/\partial\widetilde{r}_i < 0$ at the socially optimal level $\widetilde{r}_i^*$. By

[13] However, regulators assume that banks hold a well diversified credit portfolio and may impose additional capital charges if this is not the case (BIS, 2005).

contrast, bank $i's$ costs are $C_i + \tau(\widetilde{r}_i)$ and the marginal effect of an increase in $\widetilde{r}_i$ on bank i is hence $\partial C_i/\partial\widetilde{r}_i + \partial\tau_i/\partial\widetilde{r}_i$. It follows that at $\widetilde{r}_i^*$, bank i's welfare is increasing in $\widetilde{r}_i$. Hence, independent of the diversification costs τ, banks will choose a level of diversification that is larger than the socially optimal one.

Proposition 3 has a bearing on a variety of regulatory issues. For example, a main challenge for regulation has in recent years emerged from credit derivatives.[14] Regulators have so far been considering these instruments favorably, precisely on the grounds that they allow for diversification (BIS, 2004). As credit derivatives mainly seem to reallocate risks within the more fragile banking sector,[15] our analysis provides a cautioning note on their credentials. This may be important for their overall assessment since credit derivatives have also other downsides (for example, they create complex linkages among financial institutions, to mention one concern).

Diversification incentives aside, a core objective of regulation is to contain bank risk taking, which is justified by the externalities arising from bank failure. An interesting question is of whether regulators should apply different standards for diversified and undiversified banks. As in our analysis the amount invested in assets is given, bank risk taking is determined by their capital structure, where a higher amount of deposit financing implies more bank risk, as it raises the minimum return needed to avoid a bank run (equation (6)).

Figure 4 demonstrates the externality associated with increased deposits. Starting from a symmetric situation where $d_1 = d_2 = d$, it shows that an increase in the fraction of deposits at bank 1 by $\triangle d$ leads to an outward shift of $y_1(x)$ (this can also be appreciated from equation 6). As a result, the area where bank 2 fails jointly with bank 1 rather than alone increases (area A). Thus, deposit taking poses a negative externality on the other bank.

Optimal regulation has to address this externality. The question arises then of how

[14] The market for credit derivatives is developing rapidly. Only introduced ten years ago, their notional outstanding amount is predicted to reach US$ 8,200bn in 2006 and growth is expected to continue at a similar pace (BBA, 2004).

[15] Around 90% of the outstanding amount is held by banks (Fitch, 2003) and ECB (2004) estimates that the bulk of credit derivatives deals are bank-to-bank (e.g., 80% for Germany).

the latter depends on a bank's level of diversification. From Figure 4 we have that the probability of a systemic crisis at bank 2 is

$$\pi_2^S = \int_0^{\xi} y_2(x)dx + \int_{\xi}^{x_1(0)} y_1(x)dx$$

where ξ is implicitly defined by $y_1(\xi) = y_2(\xi)$. Taking the derivative with respect to d_1 gives

$$\frac{\partial \pi_2^S}{\partial d_1} = \int_{\xi}^{x_1(0)} \frac{\partial y_1(x)}{\partial d_1} dx + (y_2(\xi) - y_1(\xi))\frac{\partial \xi}{\partial d_1} + y_1(x_1(0))\frac{\partial x_1(0)}{\partial d_1}$$

From $y_1(\xi) = y_2(\xi)$ and $y_1(x_1(0)) = 0$, and using that $y_1(x) = (1 + 1/\widetilde{r}_1)d_1 - (1/\widetilde{r}_1)x$ (the equivalent of equation 6 for when d_1 and d_2 are not identical) we obtain

$$\frac{\partial \pi_2^S}{\partial d_1} = \int_{\xi}^{x_1(0)} (1 + 1/\widetilde{r}_1)dx$$

Because of symmetry in equilibrium ($d_1 = d_2 = d$), it follows that $\xi = d$ and $x_1(0) = (1 + \widetilde{r}_1)d$. Hence, the derivatives simplifies to $\partial \pi_2^S/\partial d_1 = (1 + \widetilde{r}_1)d$. Since the overall probability of a run on bank 2, π_2, is independent of d_1 we have that $\partial \pi_2^I/\partial d_1 = -\partial \pi_2^S/\partial d_1$. Thus the impact of d_1 on the costs at bank 2 is

$$\partial C_2/\partial d_1 = (q - 1)(1 + \widetilde{r}_1)d$$

showing that the externalities from risk taking increase with a bank's degree of diversification.[16] Intuitively, this is again because when a bank is more diversified, it is also more similar to the other bank. Higher risk has then a larger impact on the other bank, for the reason that both banks are then more likely to be in difficulties at the same time.[17] It follows that

Proposition 4 *Regulators should discourage risk taking more when banks are diversified.*

As diversification is likely to have increased in recent years, optimal charges for bank risk taking have thus risen. This holds in particular for large U.S. banks, whose similarities have increased substantially, as pointed out earlier (Group of Ten, 2001).

[16]This result is diametrical to the one obtained in Wagner (2006). There, higher diversification lowers externalities because the resulting increase in similarities reduces banks' ability to rely on interbank insurance, which mitigates the impact of imperfections in the interbank market.

[17]There may also be other reasons for why higher similarities, ceteris paribus, cause larger externalities. For example, Acharya (2001) presents a model where this is because correlated banks benefit from a 'too many to fail' guarantee.

5 Concluding Remarks

Diversification gains are typically seen as a main benefit of the ongoing transformation of the financial sector. By contrast, this paper has shown that diversification in the financial system can be undesirable. This is because diversification increases the similarities among institutions by exposing them to the same risks, which can make financial crises both more costly and more likely. Financial Institutions have also an incentive to diversify beyond the socially optimal level, as they do not internalize the costs of increased similarities for other institutions. As evidence indicates that the similarities among large (and thus probably systemically more relevant) institutions have indeed increased substantially during the last decade, the diversification brought about by recent developments in the financial system may in fact have been costly.

Appendix A: Endogenizing the Capital Structure

We assume now that the return on a bank's portfolio depends on whether its manager exercises effort at date 1. More, specifically we assume that only when effort is undertaken, asset returns are distributed on $[0, 1]$ (and we thus have a density $\phi = 1$ and mean $1/2$). If the manager does not exercise effort (for instance, if he does not sufficiently screen and monitor the portfolio), returns are distributed on $[0, e]$, with $e < 1$. The expected value of the asset falls then to $e/2$ and the density is $\phi^e = 1/e$. Effort is costly in that it causes a disutility a to the manager.

For simplicity, we assume that the manager cannot be motivated by monetary incentives. However, he has private benefits b from operating the bank until date 3. With equity and ordinary debt, a bank is never liquidated at date 2 as it is optimal for owners and debt holders to avoid costly liquidation of assets by renegotiating claims (for a detailed discussion of the ensuing arguments, see, for example, Diamond and Rajan, 2000). Hence, there is no incentive for the manager to exert effort at date 1.

By contrast, demand deposits constitute a non-negotiable claim: because of their first-come-first-serve nature, it is individually optimal for depositors to run whenever they fear that they will not be paid in full, thus forcing liquidation of the bank. This provides an incentive for the bank manager to achieve high asset returns (such a role for deposits in reducing agency problems within the bank has been emphasized, e.g., in Calomiris and Kahn, 1991, and Flannery, 1996).

The bank manager's expected pay-off when effort is exercised is $(1 - \pi)b - a$, i.e., the expected benefits from continuation at date 2 when effort is induced, net of effort costs a. π is probability of a bank run when assets are distributed on $[0, 1]$, as given by (10). The expected pay-off when effort is not exercised is $(1 - \pi^e)b$, where π^e is the probability of bank failure when asset returns are distributed on $[0, e]$. Consider Figure 2 and suppose that $y_1(0), x_2(0) < e$. Then, in the areas where banks fail (i.e., areas B,C and D) the densities for x and y are $\phi^e(x) = \phi(x)/e$ and $\phi^e(y) = \phi(y)/e$. It follows that

$$\pi^e = \pi \frac{\phi^e(x)\phi^e(y)}{\phi(x)\phi(y)} = \pi/e^2$$

In order for effort to be induced, we hence need

$$(1-\pi)b - a \geq (1-\pi/e^2)b \tag{13}$$

Rearranging (13) gives

$$\pi \geq \frac{a}{b}\frac{e^2}{1-e^2}$$

Hence, since the probability of a bank run π is increasing in d (equation 10), a minimum level of deposits is needed to induce effort. As deposits are costly because they cause runs, banks never set the level of deposits above this minimum level. Thus, the optimal capital structure d^* that induces effort fulfills

$$\pi(d^*) = \frac{a}{b}\frac{e^2}{1-e^2} \tag{14}$$

Note that holding liquidity (which we have ruled out from the start) is not optimal for banks as it both reduces their portfolio return (presuming that the risky asset has a higher expected return) and the threat of a bank run (banks would be better off reducing the level of deposits, which would also have the effect of making bank runs less likely but would not lower the profitability of bank's portfolio).

We next derive a condition under which bank owners find it optimal to induce effort. Bank's return from inducing effort is $1/2 - \pi^I c(1) - \pi^S c(2)$, that is the expected return on its assets net of liquidation costs. Since $c(2) > c(1)$ we have that the bank return exceeds $1/2 - \pi^I c(2) - \pi^S c(2) = 1/2 - \pi c(2)$. When effort is not induced, there is no rationale for deposits. Deposits are then optimally set to zero. Consequently, there are no bank runs and the bank's expected return is $e/2$. A sufficient condition for the optimality of inducing effort is hence $1/2 - \pi c(2) > e/2$. Substituting for π using equation (14) and rearranging gives

$$\frac{a}{b} < \frac{(1-e)(1-e^2)}{2c(2)e^2} \tag{15}$$

This condition is, for example, fulfilled for sufficiently small private effort costs a.

Recall that we have assumed that $y_1(0), x_2(0) < e$. Using (7) and applying symmetry ($\widetilde{r}_1 = \widetilde{r}_2 = \widetilde{r}$) this condition writes

$$(1+1/\widetilde{r})d < e \tag{16}$$

(16) is also fulfilled for sufficiently small effort costs a. This because from equation (14) we have that the π that is required to induce effort is arbitrarily small for small a. This, in turn, implies by (10) that also d becomes arbitrarily small and thus (16) holds. We presume from now on that both conditions (15) and (16) are fulfilled and show that Proposition 1 and 2 still hold.

Proposition 1: Besides the effects that have already been analyzed in the main text (that is, when d is given), diversification has now also indirect effects through its impact on banks' capital structures. This is because an increase in $\widetilde{r}$ lowers the probability of crises π (equation 10). As a result, the bank run threat provided by the existing deposits is not sufficient anymore to induce effort ((14) does not hold anymore). Given that inducing effort is optimal, banks have to increase their deposits in order to restore the liquidation threat. This in turn reduces welfare for investors at both banks (compared to the situation without the effort problem), as it implies higher liquidation costs. By contrast, managerial welfare is not affected. Using equation (14), a manager's equilibrium pay-off is

$$(1-\pi)b - a = (1 - \frac{a}{b}\frac{e^2}{1-e^2})b - a$$

and only depends on the primitives of the model. It follows that diversification is more costly than in the absence of the effort problem and, hence, the optimal degree of diversification is lower.

Proposition 2: The direct externality from diversification (for given d) already analyzed is still present. There is now also a second externality arising from increased deposit taking following diversification: a higher level of deposits at bank 1, for instance, reduces welfare at bank 2 because it makes systemic crises more likely (this externality is analyzed more formally in Section 4; see also Figure 4). Thus, diversification still poses negative externalities and banks will hence overdiversify in equilibrium.

Appendix B: Contagion

We assume now that in the range where the occurrence of a bank run depends on depositors' beliefs, the latter are such that depositors always run. In particular, it is assumed that each depositor believes that other depositors run, whenever this is rational. The equilibrium outcomes from the viewpoint of bank 2 are then as follows (equivalent considerations apply to bank 1):

$v_2 < d + c(1)$: Bank 2 fails regardless of whether bank 1 fails. Since $v_2 - c(1) < d$ then, bank 2 always becomes insolvent when depositors at bank 2 run. Hence, it is rational for a depositor at bank 2 to believe that other depositors at the bank will run. Since such a run leads to insolvency, a depositor with such beliefs will run. Hence all depositors at bank 2 run, leading to insolvency of the bank.

$v_2 \geq d + c(2)$: Bank 2 survives regardless of whether bank 1 fails. Since $v_2 - c(2) \geq d$, even if all depositors at both banks run, bank 2 will remain solvent. Given their weak preference for not running, depositors will thus not run on bank 2.

$d + c(1) \leq v_2 < d + c(2)$: Bank 2′s survival depends on whether bank 1 fails. If $v_1 \geq d + c(2)$, bank 2 survives. Since $v_1 - c(2) \geq d$ then, runs at bank 1 cannot occur (as shown above for bank 2). Hence, the liquidation costs in a run can be at most $c(1)$. Thus, even if all depositors at bank 2 run, the bank is still solvent (as $v_2 - c(1) \geq d$). Hence, there will be no run at bank 2. By contrast, if $v_1 < d + c(2)$, then bank 2 fails. Since $v_1 - c(2) < d$ and $v_2 - c(2) < d$ then, if depositors at both banks run, both banks will become insolvent as liquidation costs will be $c(2)$. It is hence rational for a depositor at bank 2 to believe that all other depositors will run. Thus, depositors will run on bank 2, leading to its insolvency (similarly, there will also be a run at bank 1).

Note that the equilibrium outcomes are consistent with several equilibrium selection techniques. First of all, it is easy to see that the same outcomes arise when depositors play *risk dominant* strategies (Harsanyi and Selten, 1988). This is because in the cases where runs occur, the bank always becomes insolvent. A depositor would then receive nothing if he waits until date 3. Since there is no cost of running, any (strictly) positive probability attached to a run occurring will hence make it optimal for a depositor to run as well. This equivalence to risk dominance is appealing as it has been shown that learning models often converge to the risk dominant equilibrium (Kandori, Mailath, and Rob, 1993). Sunspot driven equilibria can also be interpreted in terms of risk dominance (Ennis, 2003, p.66). Finally, when investors' signals about the fundamentals are imprecise the risk dominant equilibrium is typically obtained as well (as shown for the two player case in Carlsson and van Damme, 1993).

A main difference to the previously considered equilibrium is that there is now contagion in the banking sector as a failure of one bank can lead to the failure of the other bank. This is because a run at one bank increases the liquidation costs anticipated by depositors at the other bank and can thus make them run as well.

As before, we can characterize the equilibrium in terms of minimum return functions $y_i(x)$. However, there are now two different thresholds $y_i^{c(1)}(x)$ and $y_i^{c(2)}(x)$ for each bank, corresponding to liquidation costs $c(1)$ and $c(2)$, respectively. Similarly to equation (6), the minimum return functions are

$$y_1^{c(1)}(x) = (1 + 1/\widetilde{r}_1)(d + c(1)) - (1/\widetilde{r}_1)x \text{ and } y_1^{c(2)}(x) = (1 + 1/\widetilde{r}_1)(d + c(2)) - (1/\widetilde{r}_1)x$$
$$y_2^{c(1)}(x) = (1 + \widetilde{r}_2)(d + c(1)) - \widetilde{r}_2 x \text{ and } y_2^{c(2)}(x) = (1 + \widetilde{r}_2)(d + c(2)) - \widetilde{r}_2 x \quad (17)$$

Figure 5 shows these functions. The depicted intersections with the axes are given by

$$y_1^{c(1)}(0) = (1+1/\widetilde{r}_1)(d+c(1)) \text{ and } y_2^{c(2)}(0) = (1+\widetilde{r}_2)(d+c(2)) \text{ and } x_1^{c(2)}(0) = (1+\widetilde{r}_1)(d+c(2)) \quad (18)$$

The equilibrium crisis outcomes can be expressed as follows

- $y < y_1^{c(1)}(x)$ and $y \geq y_2^{c(2)}(x)$: individual crisis at bank 1 (Area A)
- $y \geq y_1^{c(2)}(x)$ and $y < y_2^{c(1)}(x)$: individual crisis at bank 2 (Area B)
- $y < y_1^{c(2)}(x)$ and $y < y_2^{c(2)}(x)$: systemic crisis (area southwest of intersection of $y_1^{c(2)}(x)$ and $y_2^{c(2)}(x)$)

We show first that Proposition 1 still holds and derive the optimal degree of diversification. In doing so, we focus on bank 1 (which minimizes notation). From Figure 5 we have that the probability of a systemic crisis is

$$\begin{aligned}\pi^S &= (d+c(2))^2 + (d+c(2))(y_2^{c(2)}(0) - (d+c(2))/2 + (d+c(2))(x_1^{c(2)}(0) - (d+c(2))/2 \\ &= (d+c(2))^2(1+\frac{\widetilde{r}_1+\widetilde{r}_2}{2}) \qquad (19)\end{aligned}$$

which simplifies to $\pi^S = (d+c(2))^2(1+\widetilde{r})$ for $\widetilde{r}_1 = \widetilde{r}_2$ (symmetry). Thus, as before, we have $\pi^{S\prime}(\widetilde{r}) = (d+c(2))^2 > 0$, i.e., diversification increases the probability of a systemic crisis.

We derive next the probability of an individual crisis at bank 1. Note that Figure 5 presumes that $y_1^{c(1)}(0) \geq y_2^{c(2)}(0)$. Using (18), this condition can be rearranged to

$$\widehat{r} \leq \frac{d+c(1)}{d+c(2)} \qquad (20)$$

If (20) is fulfilled then π_1^I is given by area A in Figure 5

$$\pi_1^I = \int_0^{\xi} (y_1^{c(1)} - y_2^{c(2)})dx \qquad (21)$$

where ξ is implicitly defined by $y_1^{c(1)}(\xi) = y_2^{c(2)}(\xi)$. Taking derivative wrt. $\widetilde{r}$ and using $y_1^{c(1)}(\xi) = y_2^{c(2)}(\xi)$ gives

$$\pi_1^{I\prime}(\widetilde{r}) = \int_0^{\xi} (-(d+c(1)-x)/r^2 - (d+c(2)-x))dx$$

From Figure 5 one can see that $x < d+c(1)$ and $x < d+c(2)$ for $x \leq \xi$. Hence, the integral is negative and an increase in $\widetilde{r}$ reduces the probability of an individual crisis ($\partial\pi_1^I/\partial r < 0$). Diversification then poses a similar trade-off as before: it makes individual crises less likely, but this comes at the expense of increasing systemic crises.

If condition (20) is not fulfilled (i.e., if $y_1^{c(1)}(0) < y_2^{c(2)}(0)$), there are no individual crisis: $\pi_1^I = 0$. Diversification does then not have any benefits. It follows immediately that diversification which exceeds $\frac{d+c(1)}{d+c(2)}$ is never optimal. Therefore, Proposition 1 holds, as $\frac{d+c(1)}{d+c(2)} < 1$. Interestingly, it also follows that diversifying beyond $\frac{d+c(1)}{d+c(2)}$ would even increase the overall probability of banking failure, as it increases the probability of a systemic crisis but leaves the probability of an individual crisis unchanged.

We derive now the expression for the optimal degree of diversification. The latter solves

$$\widetilde{r}^* = \min(\arg\min_{\widetilde{r}}(\pi_1^I c(1) + \pi_2^I c(1) + 2\pi^S c(2)), \frac{d+c(1)}{d+c(2)})$$

Using symmetry ($\pi_1^I = \pi_2^I$), the FOC for $\widetilde{r}$ for the first term in the minimum operator is $q\pi^{S\prime}(\widetilde{r}) = -\pi_1^{I\prime}(\widetilde{r})$. Making use of the expressions for $\pi^{S\prime}(\widetilde{r})$ and $\pi_1^{I\prime}(\widetilde{r})$ the FOC writes

$$q(d + c(2))^2 = \int_0^{\xi} ((d + c(1) - x)/r^2 - (d + c(2) - x))dx$$

Using $y_1^{c(1)}(\xi) = y_2^{c(2)}(\xi)$ to solve for ξ gives

$$\xi = \frac{(1 + 1/\widetilde{r})(d + c(1)) - (1 + \widetilde{r})(d + c(2))}{1/\widetilde{r} - \widetilde{r}}$$

Substitution into the FOC and solving for $\widetilde{r}$ yields

$$\widetilde{r} = \frac{1}{\sqrt{2q-1}} \frac{d + c(2)}{d + qc(2)}$$

Thus, the optimal degree is given by

$$\widetilde{r}^* = \min(\frac{1}{\sqrt{2q-1}} \frac{d + c(2)}{d + qc(2)}, \frac{d + c(1)}{d + c(2)})$$

As $\frac{d+c(2)}{d+qc(2)} < 1$, the first term is lower than the optimal degree without contagion ($= \frac{1}{\sqrt{2q-1}}$ from equation (11)). It follows that $\widetilde{r}^*$ is lower than in absence of contagion.

Second, we show that also Proposition 2 holds. To this end, consider the impact of an increase in $\widetilde{r}_2$ on bank 1, starting from $\widetilde{r}_1 = \widetilde{r}_2$. From (19) we have that the probability of a systemic crisis increases, as $\pi^{S\prime}(\widetilde{r}_2) = (d + c(2))^2/2 > 0$. Suppose that condition (20) holds. From (21) we find that the impact on the probability of an individual crisis is

$$\pi_1^{I\prime}(\widetilde{r}_2) = \int_0^{\xi} -y_2^{c(2)\prime}(\widetilde{r}_2)dx = -\int_0^{\xi}(d+c(2)-x)dx > -\int_0^{d+c(2)}(d+c(2)-x)dx = -(d+c(2))^2/2$$

where we have used $\xi < d + c(2)$. We thus have that $\pi^{S\prime}(\widetilde{r}_2) > -\pi_1^{I\prime}(\widetilde{r}_2)$, that is the increase in the probability of a systemic crisis outweighs the reduction in the probability of an individual crisis. Obviously, this also holds if (20) is not fulfilled, as then $\pi_1^{I\prime}(\widetilde{r}_2) = 0$. Thus, an increase in $\widetilde{r}_2$ always poses a negative externality on bank 1. It follows, that the market equilibrium exhibits excessive diversification.

Appendix C: Generalized Distribution Function

Proposition 1: From Figure 2 we have that the probability of failure at bank 2 is

$$\pi_2(\widetilde{r}) = \int_0^{x_2(0)} (\int_0^{y_2(x)} \phi(x)\phi(y)dy)dx$$

Taking derivative wrt. $\widetilde{r}$ and evaluating at $\widetilde{r} = 1$ gives

$$\begin{aligned}
\frac{d\pi_2}{d\widetilde{r}}(\widetilde{r} &= 1) = \int_0^{x_2(0)} (d-x)\phi(x)\phi(y_2(x))dx - \frac{d}{\widetilde{r}^2}\int_0^{y_2(x_2(0))} \phi(x_2(0))\phi(y)dy \\
&= \int_0^{2d} (d-x)\phi(x)\phi(2d-x)dx
\end{aligned}$$

where use has been made of equations (6) and (7). Defining $f(x) = (d-x)\phi(x)\phi(2d-x)$ and noting that $f(x) = -f(2d-x)$ we can further write

$$\begin{aligned}
\frac{d\pi_2}{d\widetilde{r}}(\widetilde{r} &= 1) = \int_0^{2d} f(x)dx = \int_0^{d} f(x)dx + \int_d^{2d} f(x)dx \\
&= \int_0^{d} f(x)dx - \int_d^{2d} f(2d-x)dx = \int_0^{d} f(x)dx - \int_0^{d} f(x)dx = 0
\end{aligned}$$

Thus, as before, the marginal benefits from diversification are zero at $\widetilde{r} = 1$. The probability of a systemic crisis also remains increasing in $\widetilde{r}$ (areas A and C in Figure 3 are always associated with a positive probability mass because of $\phi(x), \phi(y) > 0$). Hence, at $\widetilde{r} = 1$ there are again no marginal benefits from diversification but there are marginal costs. It follows that $\widetilde{r} < 1$ is still optimal.

Proposition 2: The validity of Proposition 2 can be directly appreciated by considering Figure 3. More diversification at bank 1 impacts bank 2 by causing systemic crises in the area C. Since $\phi(x), \phi(y) > 0$, this area has a positive probability mass. Hence, there is a negative externality and banks overdiversify in equilibrium.

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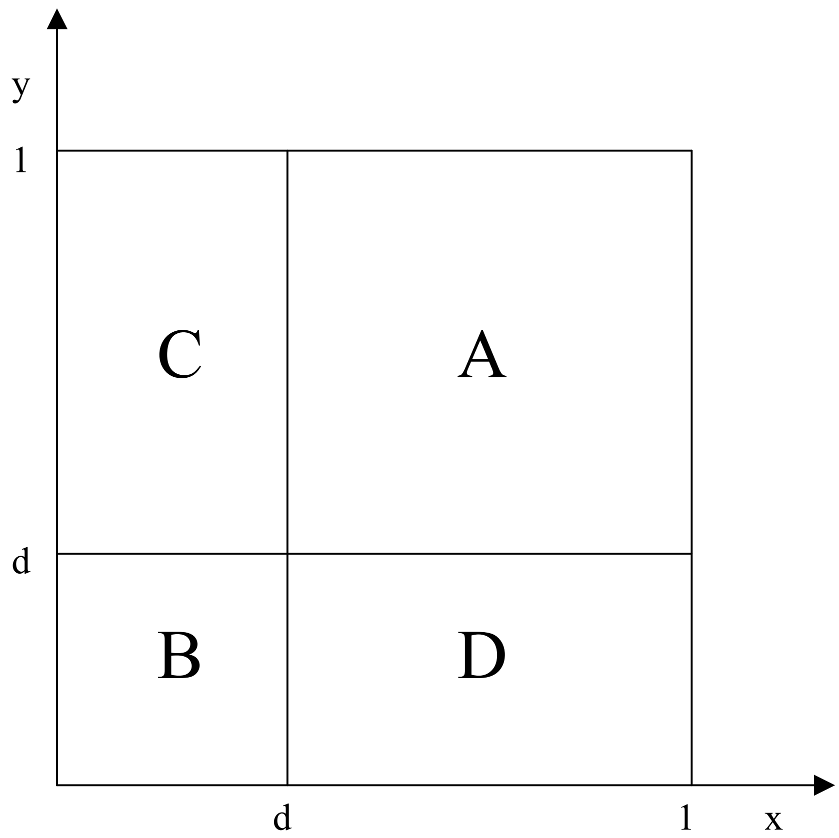
Figure 1: The Undiversified Banking Sector



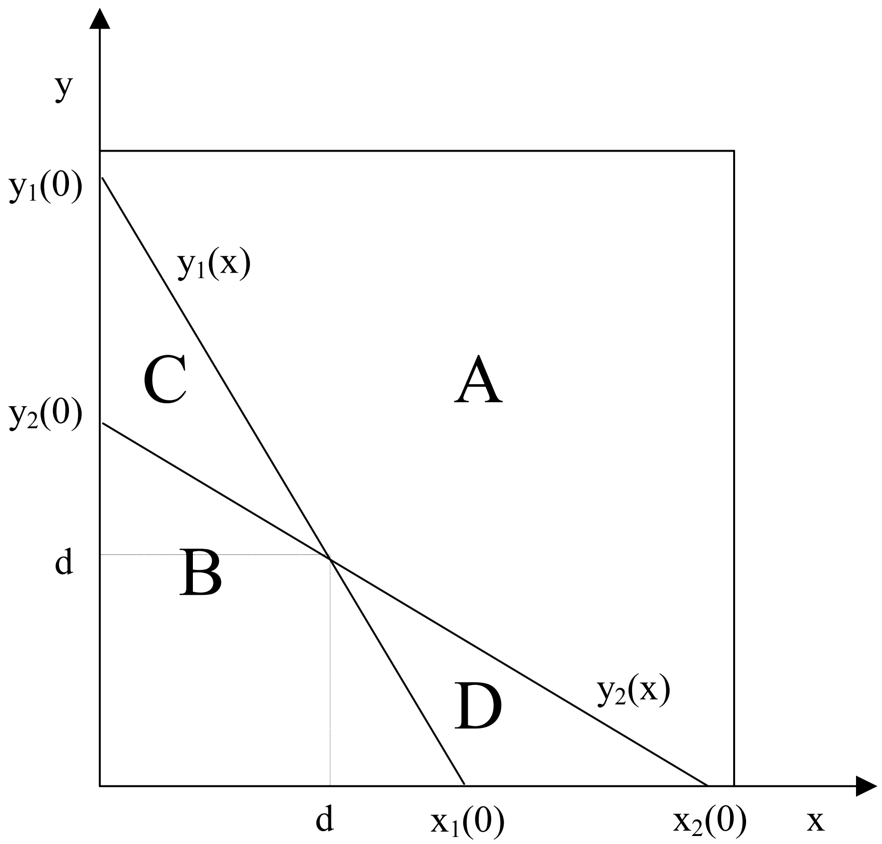
A: no crisis

B: systemic crisis

C: individual crisis at bank 1

D: individual crisis at bank 2

Figure 2: A (Partially) Diversified Banking Sector



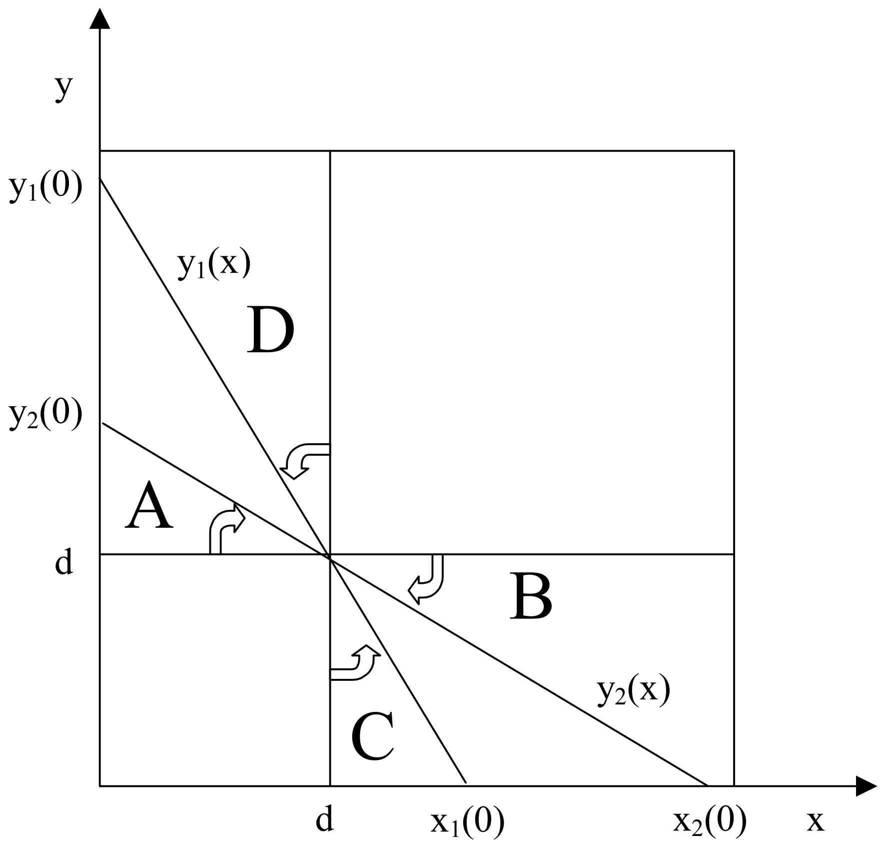
A: no crisis

B: systemic crisis

C: individual crisis at bank 1

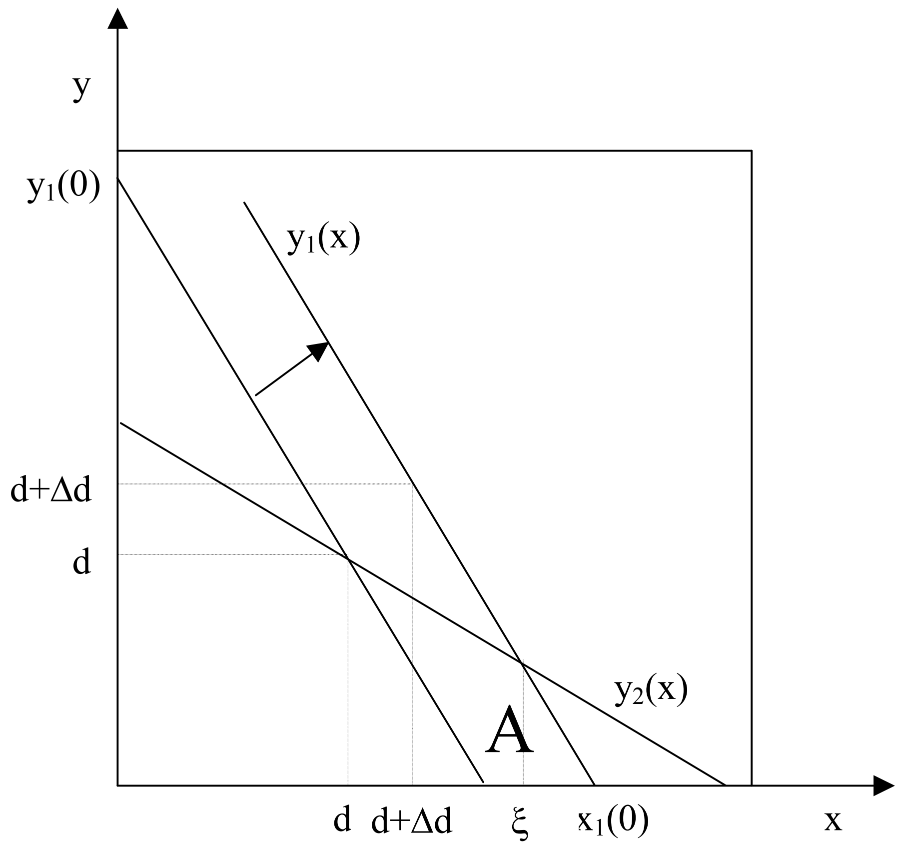
D: individual crisis at bank 2

Figure 3: The Impact of an Increase in Diversification at Both Banks



Impact on Bank 1:	Impact on Bank 2:
C: no crisis $\Rightarrow$ systemic crisis	A: no crisis $\Rightarrow$ systemic crisis
D: individual crisis $\Rightarrow$ no crisis	B: individual crisis $\Rightarrow$ no crisis
A: individual crisis $\Rightarrow$ systemic crisis	C: individual crisis $\Rightarrow$ systemic crisis

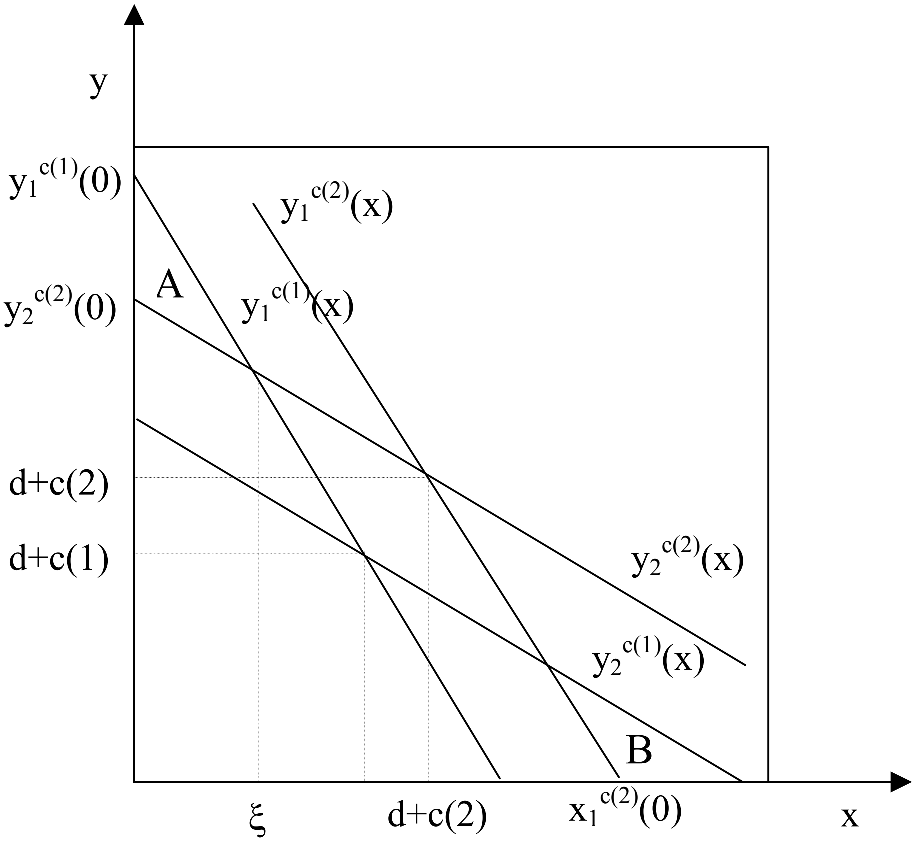
Figure 4: The Impact of an Increase in the Deposits at Bank 1



Impact on Bank 2:

A: individual crisis $\Rightarrow$ systemic crisis

Figure 5: Contagion



A: individual crisis at bank 1

B: individual crisis at bank 2